Basic Exercises
Micro Economics
AKG
# Table of Contents

**MICRO ECONOMICS**

**Budget Constraint** ................................................................. 4  
Practice problems ................................................................. 4  
Answers ................................................................. 4  

**Supply and Demand** ................................................................. 7  
Practice Problems ................................................................. 7  
Answers ................................................................. 7  
Additional problems ................................................................. 9  
Answers ................................................................. 10  

**Elasticity and Taxes** ................................................................. 13  
Practice Problems (Elasticity) ................................................................. 13  
Answers ................................................................. 13  
Practice problems (Elasticity and Taxes) ................................................................. 14  
Answers ................................................................. 14  

**Demand and Applications** ................................................................. 16  
Practice Problems (Demand) ................................................................. 16  
Answers ................................................................. 16  
Practice problems (Deriving Demand and Applications) ................................................................. 16  
Answers ................................................................. 18  

**General Equilibrium** ................................................................. 19  
Practice Problems(1) ................................................................. 19  
Answers ................................................................. 19  
Practice Problems(2) ................................................................. 20  
Answers ................................................................. 21  

**Production** ................................................................. 22  
Practice problems ................................................................. 22  
Answers ................................................................. 22  

**Cost** ................................................................. 24  
Practice Problems ................................................................. 24  
Answers ................................................................. 25
Perfect Competition ............................................................... 26
Practice problems ............................................................... 26
Answers ............................................................... 26
Monopoly ............................................................... 27
Practice Problems ............................................................... 27
Answers ............................................................... 28
Cournot Duopoly ............................................................... 29
Practice problems ............................................................... 29
Answers ............................................................... 29
Game Theory ............................................................... 30
Practice Problems ............................................................... 30
Answers ............................................................... 31
Practice Problems

1. a) Graph the budget constraint for a consumer who can buy either of two goods, X and Y. The price of good X is $10 per unit, and the price of good Y is $5 per unit, and the consumer has $200 to spend. (Put good X on the horizontal axis and good Y on the vertical axis).

b) What is the slope of the budget constraint?

c) Suppose the price of good Y increases to $10. Use a graph to show how the budget constraint changes.

d) Suppose the price of good Y is $5, and the price of good X increases to $20. Repeat the problem above. Use a graph to show how the budget constraint changes.

e) Now suppose the prices return to their original value, but the consumer gets a raise that allows them to have $300 to spend. Use a graph to show how the budget constraint changes.

Answers

1.

a)
b) The slope of the budget constraint is -2.

c)

d)
Supply and Demand

Practice Problems

1. Guitars and guitar strings are complements. If the price of guitars rises, what can we expect to happen to the price of guitar strings?

2. If the actual price of a good is above the equilibrium price, what will likely happen to the price, the quantity demanded, and the quantity supplied?

3. The demand for a particular product is given by \( Q_d = 100 - 10P \). The supply for that product is given by \( Q_s = 20P - 50 \). Calculate the equilibrium price and quantity of this good.

4. Using the information in question 4, suppose the price were $3. Calculate the excess demand or excess supply of the product.

5. Suppose canned tuna is an inferior good. If consumers' incomes rise, what would you expect would happen to the equilibrium price and equilibrium quantity of canned tuna?

6. If the cost of producing a product increases, what will likely happen to the equilibrium price and quantity of the product?

7. Describe the difference between a change in demand and a change in quantity demanded.

8. What do you predict will happen to the equilibrium price and quantity of a product if the price of a substitute for that product increases at the same time a technological breakthrough makes the product cheaper to produce?

9. The equilibrium price is a price where firms sell exactly how much they plan to sell, and consumers buy exactly how much they plan to buy (the quantity demanded is equal to the quantity supplied). To find the equilibrium price, we must solve two equations simultaneously for the two variables, \( P \) (price) and \( Q \) (quantity). Solve the following two equations for the variables \( P \) and \( Q \).

\[
\begin{align*}
a) \quad Q &= 4 - 2P; \quad Q = 2P \\
b) \quad Q &= 25 - 2P; \quad Q = P - 8 \\
c) \quad Q &= 26 - 2P; \quad Q = 2P - 10 \\
d) \quad Q &= 10 - 3P; \quad Q = 2P - 5
\end{align*}
\]

Answers

1. If the price of guitars rises, that will decrease the demand for guitar strings. A decrease in demand for guitar strings will lead to a decrease in the equilibrium price of guitar strings.
2. If the actual price of a good is above the equilibrium price, then there will be excess supply of that product ($Q_s > Q_d$). There will be a tendency for the price of the product to drop. As this happens, the quantity supplied will also decrease, and the quantity demanded will increase until $Q_d = Q_s$.

3. Equilibrium is where $Q_d = Q_s$. Setting $Q_d = Q_s$, we get $100 - 10P = 20P - 50$. Solving for $P$, $P = 5$. Substituting $P = 5$ back into either the supply or the demand equation and solving for $Q$, $Q = 50$. Note that a good way to check your answer for $P$ is to substitute it back into both the supply and the demand equation and make sure you get the same answer. If not, you made a mistake somewhere!

4. At a price of $3$, $Q_d = 70$, and $Q_s = 10$. So there is excess demand of 60 ($Q_d - Q_s$).

5. If consumers' incomes rise, this will decrease the demand for an inferior good. A decrease in demand will cause both the equilibrium price and the equilibrium quantity to fall.

6. An increase in the cost of production will decrease the supply of the product. This will cause the equilibrium price to rise, and the equilibrium quantity to fall.

7. A change in demand is caused by a change in something OTHER than the price of the product (such as a change in income, preferences, etc). It is represented by a shift of the demand curve. An increase in demand is a shift to the right of the demand curve, and a decrease in demand is a shift to the left of the demand curve.

A change in quantity demanded is caused by a change in the price of the good, and is represented by a movement ALONG a demand curve.

8. Here we are dealing with a simultaneous increase in demand and an increase in supply. To answer this question, it is useful to break it up into 2 parts. First, draw an increase in demand and you will see that $P$ and $Q$ both rise. Second, draw an increase in supply and you will see that $P$ falls and $Q$ rises. Now put these results together—we know for sure that $Q$ will rise, but we cannot say without more information what will happen to $P$.

9.a) $P=1$; $Q=2$; b) $P=11$; $Q=3$; c) $P=9$; $Q=8$; d) $P=3$; $Q=1$
Additional problems

You are given

Demand:  \( Q_d = 1000 - 20P + 10P_s - 10P_c + 6Y \)

Supply:  \( Q_s = -180 + 40P - 40C \)

Where:

\( Q_d \) = quantity demanded

\( Q_s \) = quantity supplied

\( P \) = the price of the good

\( P_s \) = the price of a substitute

\( P_c \) = the price of a complement

\( Y \) = the consumer’s income

\( C \) = the marginal cost of producing the good.

Suppose \( P_s = 5; P_c = 8, Y = 25, C = 20. \)

1) Calculate the equilibrium price and quantity in this market. (Hint: remember that equilibrium occurs at the price where \( Q_d = Q_s \)).

2) Now suppose that the price of a substitute rises to 11.
   a) Calculate the new equilibrium price and quantity.
   b) How does the price and quantity compare to the original price and quantity?
   c) How does the increase in the price of the substitute affect the demand for the good?

3) Now suppose \( P_s \) is at its original value, and \( P_c \) rises to 20.
   a) Calculate the new equilibrium price and quantity.
   b) How does the price and quantity compare to the original price and quantity?
   c) How does the increase in the price of the complement affect the demand for the good?
4) Now suppose $P_S$ and $P_C$ are at their original values, and $Y$ increases to 45.

a) Calculate the new equilibrium price and quantity.

b) How does the price and quantity compare to the original price and quantity?

c) How does the increase in income affect the demand for the good?

d) Is this a normal good or an inferior good?

5) Now using the original demand, suppose the marginal cost of production ($C$) decreases to 17.

a) Calculate the new equilibrium price and quantity.

b) How does the price and quantity compare to the original price and quantity?

c) How does the decrease in the marginal cost of production affect the supply of the good?

Answers

1) $Qd = 1000 - 20P + 10P_S - 10P_C + 6Y$

$Qs = -180 + 40P - 40C$

Suppose $P_S = 5$; $P_C = 8$, $Y = 25$, $C = 20$

$Qd = 1000 - 20P + 10(5) - 10(8) + 6(25) = 1120 - 20P$

$Qs = -180 + 40P - 40(20) = 40P - 980$

$Qs = Qd \rightarrow$

$1120 - 20P = 40P - 980 \rightarrow$

$2100 = 60P \rightarrow$

$P = 35$

$Qd = 420$

$Qs = 420$

2) Suppose $P_S = 11$

a) $Qd = 1180 - 20P$ (new demand)
\[ Q_s = 40P - 980 \text{ (original supply)} \]

**Equilibrium →**

\[ 1180 - 20P = 40P - 980 → \]

\[ 2160 = 60P → \]

\[ P = 36 \]

\[ Q_d = 460 \]

\[ Q_s = 460 \]

*b) The new equilibrium price and quantity are both higher than the original equilibrium price and quantity.*

c) The increase in the price of the substitute causes an increase in the demand for the good. Graphically, this would be represented by a shift to the right of the demand curve.

**3) Now suppose** \( P_s \) **is at its original value, and** \( P_C \) **rises to 20**

*a) \( Q_d = 1000 - 20P \) (new demand)*

\[ Q_s = 40P - 980 \text{ (original supply)} \]

**Equilibrium →**

\[ 1000 - 20P = 40P - 980 → \]

\[ 1980 = 60P → \]

\[ P = 33 \]

\[ Q_d = 340 \]

\[ Q_s = 340 \]

*b) The new equilibrium price and quantity are both lower than the original equilibrium price and quantity.*

c) The increase in the price of the complement causes a decrease in the demand for the good. Graphically, this would be represented by a shift to the left of the demand curve.

**4) Now suppose** \( P_s \) **and** \( P_C \) **are at their original values, and** \( Y \) **increases to 45.**

*a) \( Q_d = 1240 - 20P \) (new demand)*

\[ Q_s = 40P - 980 \text{ (original supply)} \]

**Equilibrium →**
1240 – 20P = 40P -980 \rightarrow  \\
2220 = 60P \rightarrow  \\
P = 37  \\
Qd = 500  \\
Qs = 500  \\
b) The new equilibrium price and quantity are both higher than the original equilibrium price and quantity.  

c) The increase in income causes an increase in the demand for the good. Graphically, this would be represented by a shift to the right of the demand curve.  

d) Since the increase in income causes an increase in demand, this is a normal good.

5) Back to original demand, suppose C decreases to 17 

a) Qd = 1120 – 20P (original demand)  
Qs = 40P – 860 (new supply)  

Equilibrium \rightarrow  

1120 – 20P = 40P -860 \rightarrow  

1980 = 60P \rightarrow  

P = 33  \\
Qd = 460  \\
Qs = 460  \\
b) The new equilibrium price is lower and the equilibrium quantity is higher than the original equilibrium price and quantity.  

c) A decrease in the marginal cost of production causes an increase in the supply of the good. Graphically this would be represented by a shift to the right of the supply curve.
Practice problems

1. Calculate the Price elasticity of demand, $\epsilon$, for the following examples:

   a) Demand is given by $Q = 50 - P$ at the price of $10$.

   b) Demand is given by $Q = 100 - P$, at the price of $50$.

   c) Demand is given by $Q = 25 - .25P$, at the price of $40$.

   d) Demand is given by $Q = 20 -.1P$, at the price of $80$.

   e) Demand is given by $Q = 60 - 1/3P$, at the price of $60$.

2. Suppose the demand for crossing the Harbor Tunnel in Baltimore is given by:

   \[ Q_d = 10,000 - 1000P. \]

   a. If the toll (price) is $2$, how much total revenue is collected?

   b. What is the price elasticity of demand when the price is $2$?

   c. If the tunnel authorities want to increase their total revenue, should they increase or decrease the price from $2$?

   d) Repeat a-c above for a price of $6$.

   e) What price should the tunnel authorities set if they want to maximize their revenue? (Hint: remember that revenue is maximized when $\epsilon = -1$).

Answers

1. a) $-0.25$

   b) $-1$

   c) $-0.667$

   d) $-0.667$

   e) $-0.5$

2. a) $Q_d = 10,000 - 1000P$. If $P = 2$, then Quantity = 8000. Therefore total revenue = $P*Q = 2*8000 = 16,000$. 


b) -.25

c) Since demand is inelastic at a price of $2, they should increase the toll.

d) $2400; -1.5; Decrease

e) P = $5 will maximize revenue

Practice Problems (Elasticity and Taxes)

1. Suppose the market supply and demand for guitars in Happy Valley are given by:

Demand: \( Q = 4000 - 4P \)

Supply: \( Q = -200 + P \)

A) Calculate the equilibrium price and quantity of guitars.

B) What is the Price Elasticity of Demand at the equilibrium price and quantity?

C) What is the Price Elasticity of Supply at the equilibrium price and quantity?

For the remaining questions, suppose a tax of $10 per guitar is levied on the consumers.

D) What proportion of the tax will be paid by consumers?

E) How much will consumers pay for guitars after the tax?

F) How much will producers receive after the tax?

2. Repeat Question #1 with the following information:

Demand: \( Q = 3000 - 2P \)

Supply: \( Q = -600 + 4P \)

For Parts D-F there is a $30 per unit tax levied on consumers.

3. Suppose the price elasticity of demand, \( \varepsilon = -2 \). Also, the price elasticity of supply, \( \eta = 3 \). If a per unit tax, \( t \) is levied on consumers, what proportion of the tax will be paid by the consumers? What proportion will be paid by producers? How would your answer change if the tax were instead levied on the producers?

Answers

1. A) \( P^* = 840 \); \( Q^* = 640 \);

B) -5.25
C) 1.3125

D) The consumers will pay 1/5 (0.2) of the tax.

E) The consumers will pay the equilibrium price ($840) plus 1/5 of the $10 tax ($2), so consumers will pay $842 per guitar.

F) Producers will receive the equilibrium price ($840) minus 4/5 of the tax ($8), so producers will receive $832 per guitar.

2. A) \( P^* = 600 \quad Q^* = 1800; \)
   B) \(-\frac{2}{3}\)
   C) \(\frac{4}{3}\)

D) The consumers will pay 2/3 of the tax.

E) The consumers will pay the equilibrium price ($600) plus 2/3 of the $30 tax ($20), so consumers will pay $620 per guitar.

F) Producers will receive the equilibrium price ($600) minus 1/3 of the tax ($10), so producers will receive $590 per guitar.

3. The consumers will pay 3/5 of the tax. Producers will pay 2/5 of the tax. The results would not change if the tax were instead levied on the producers.
Practice Problems

1. Suppose Consumer preferences are given by the utility function \( U(X,Y) = XY^2 \). From this utility function,

\[
MU_X = Y^2
\]

\[
MU_Y = 2XY
\]

Suppose the consumer has $300 to spend (M = 300).

a) Derive an expression for the demand for good X.

b) Sketch the graph of this demand curve (you might want to make a table containing several different price/quantity combinations.)

Answers

\[
MRS = MRT \rightarrow \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y}.
\]

Therefore,

\[
\frac{Y^2}{2XY} = \frac{P_X}{P_Y}.
\]

So

\[
\frac{Y}{2X} = \frac{P_X}{P_Y}.
\]

Solving for Y,

\[
Y = \frac{2XP_X}{P_Y}.
\]

Substitute this into the budget constraint and solve for X. You should get

\[
X = \frac{300}{3P_X}.
\]

Since \( M = 300 \), the expression is

\[
X = \frac{300}{3P_X}.
\]

Practice Problems (Deriving Demand and Applications)

1. Suppose a consumer’s preferences are given by \( U(X,Y) = X*Y \). Thus, the marginal utility of X, \( MU_X = Y \) and the marginal utility of Y, \( MU_Y = X \). Suppose the consumer has $100 to spend and the price of good Y is $1. Sketch the price-consumption curve for the prices of \( P_X = $1 \), \( P_X = $2 \) and \( P_X = $5 \). To do this, carefully draw the budget constraints associated with each of the prices for good X, and indicate the bundle that the consumer chooses in each case. Also, be sure to label your graph accurately.

2. Suppose a consumer’s preferences are given by \( U(X,Y) = X*Y \). Suppose the price of good Y is $1 and the price of good X is $2. Sketch the income-consumption curve for the values \( M = $20 \), \( M = $60 \) and \( M = $100 \). To do this, carefully draw the budget constraints associated with each of the prices for good X, and indicate the bundle that the consumer chooses in each case. Also, be sure to label your graph accurately.

3. For these questions, refer to the diagram below.
a) Assuming the consumer has $20 to spend, identify three price/quantity combinations which are on the consumer’s demand curve for Snacks.

4. Explain what is being measured on both the vertical and horizontal axes for
   a) a price-consumption curve.
   b) an income consumption curve.
   c) a demand curve.
   d) an Engel curve.

5. Refer to the following diagram:

   In this diagram, B₁ is the original budget constraint, and B₂ is the budget constraint after the price of soda has changed.

   a) Did the price of soda increase or decrease?
b) How much does the quantity demanded of soda change as a result of the change in the price of soda?

c) How much of the change in quantity demanded is a result of the substitution effect?

d) How much of the change in quantity demanded is a result of the income effect?

e) Is soda an inferior good or a normal good?

Answers

1. All of the budget constraints should have a vertical intercept (Y-axis) of 100. When \( P_x = 1 \), the horizontal intercept should be 100, and the bundle chosen is \((X = 50, Y = 50)\). When \( P_x = 2 \), the horizontal intercept should be 50, and the bundle chosen is \((X = 25, Y = 50)\). When \( P_x = 5 \), the horizontal intercept should be 20, and the bundle chosen is \((X = 10, Y = 50)\).

2. The first budget constraint should have a vertical intercept (Y-axis) of 20 and a horizontal intercept (X-axis) of 10. The bundle chosen is \((X = 5, Y = 10)\).

The second budget constraint should have a vertical intercept (Y-axis) of 60 and a horizontal intercept (X-axis) of 30. The bundle chosen is \((X = 15, Y = 30)\).

The third budget constraint should have a vertical intercept (Y-axis) of 100 and a horizontal intercept (X-axis) of 50. The bundle chosen is \((X = 25, Y = 50)\).

3.

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.50</td>
<td>10</td>
</tr>
<tr>
<td>$1</td>
<td>7</td>
</tr>
<tr>
<td>$2</td>
<td>5</td>
</tr>
</tbody>
</table>

4. a) Quantity of good Y on the vertical axis; quantity of good X on the horizontal axis.

b) Quantity of good Y on the vertical axis; quantity of good X on the horizontal axis.

c) Price of the good on the vertical axis; quantity of the good on the horizontal axis

d) Income (amount of money to spend) on the vertical axis; quantity of the good on the horizontal axis.

5. a) Decrease; b) 10; c) 3; d) 7; e) Normal
**General Equilibrium**

*Practice Problems (1)*

1. Suppose Ann and Bill receive food and clothing. Ann has 150 units of clothing, and Bill has 100 units of clothing. Also, Ann has 75 units of food, and Bill has 75 units of food. Draw the Edgeworth Box and illustrate the initial endowment. Make sure to label your illustration carefully.

2. Define Pareto Efficiency. What condition must be met in order for an allocation to be Pareto efficient?

3. Suppose Ann’s marginal rate of substitution between food and clothing is -1/2. Also, Bill’s marginal rate of substitution between food and clothing is -2. Propose a trade between Ann and Bill that would make them both better off.

*Answers*

1.
2. Pareto Efficiency is a situation where no one can be made better off without making someone else worse off. Note that it does not necessarily imply an equitable situation. For instance, if you and I are asked to split $100, the situation where you get $100 and I get nothing is Pareto efficient. This is because in order to make me better off, some money would have to be taken away from you.

In order for an allocation in an exchange economy to be pareto efficient, the marginal rate of substitution must be equal for each person.

3. Ann is willing to give up \( \frac{1}{2} \) unit of food for 1 unit of clothing. Put another way, Ann is willing to give up 2 units of clothing for 1 unit of food. On the other hand, Bill is willing to give up 2 units of food for 1 unit of clothing. So suppose Bill gives Ann one unit of food in exchange for one unit of clothing. Both Ann and Bill would be better off.

Practice Problems (2)

Suppose there are two consumers, A and B, and two goods, X and Y. Consumer A is given an initial endowment of 25 units of good X and 10 units of good Y. Consumer B is given an initial endowment of 100 units of good X and 10 units of good Y. Consumer A’s utility function is given by:

\[ U_A(X,Y) = X^3 Y \]

And consumer B’s utility function is given by

\[ U_B(X,Y) = X^3 Y. \]

Therefore, consumer A’s marginal utilities for each good are given by:

\[ MU_X = Y^3 \]
\[ MU_Y = 3X^2 Y \]

Also, consumer B’s marginal utilities for each good are given by:

\[ MU_X = Y \]
\[ MU_Y = X \]

Initial Endowments are given by

- Person A has 40 units of good X and 20 units of good Y.
- Person B has 30 units of good X and 20 units of good Y.

Suppose \( P_X = 1 \).

a) What is the value of \( P_Y \) that will result in a competitive equilibrium?

b) How much of each good will each consumer demand in equilibrium?
Answers

a) $P_Y = 3$

b) Person A demands 25 units of good X and 25 units of good Y. Person B demands 45 units of good X and 15 units of good Y.

Tips on how to solve this problem:

Determine each consumer’s demand for each good in terms of prices and incomes. This is done by setting the MRS for each consumer equal to the MRT. Solve for one good in terms of the other, and substitute into the budget constraint.

Determine each consumer’s wealth (M) based on their initial endowment and the prices of each good. $P_X = 1$ and you are solving for $P_Y$.

Use the condition that the total demand for good X (the sum of each person’s demand for good X) must equal the total supply of good X (the sum of each person’s endowment of good X). This will allow you (after a bit of algebra) to find $P_Y$.

Once you know prices, you can calculate the actual demands for each good for each consumer.
Practice Problems

1. Suppose a firm’s production function is \( Q = L^{1/2} K^{3/4} \). Does this production function exhibit increasing, constant, or decreasing returns to scale? Explain.

2. Suppose a firm’s production function is given by \( Q = L*K \).
   - Sketch a graph of the isoquant for which the firm produces 10 units of output.
   - What is the marginal product of labor?
   - What is the marginal product of capital?
   - What is the marginal rate of technical substitution between capital and labor?

3. Suppose a firm’s production function is given in the following table

<table>
<thead>
<tr>
<th>L</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>105</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>126</td>
</tr>
<tr>
<td>8</td>
<td>128</td>
</tr>
<tr>
<td>9</td>
<td>126</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
</tr>
</tbody>
</table>

   - What is the marginal product of the 5th unit of labor?
   - With which unit of labor does diminishing marginal returns set in?
   - What is the average product of labor when the firm uses 8 units of labor?

Answers

1. This production function exhibits increasing returns to scale. To see this, note that \( f(2L,2K) = (2L)^{1/2}(2K)^{3/4} = 2^{1/2} L^{1/2} 2^{3/4} K^{3/4} = 2^{5/4} L^{1/2} K^{3/4} \). This is more than twice as much as \( L^{1/2} K^{3/4} \).

2. To graph the isoquant, simply find different combinations of \( L \) and \( K \) which yield 10 units of output (\( K*L = 10 \)). Plot these points and graph them.

The \( MP_L = K \) and the \( MP_K = L \). One way to find the \( MP_L \) is to calculate how much output changes when \( L \) changes by one unit. That is, \( f(L,K) = L*K \), and \( f(L+1,K) = K*L + K \). Thus, when \( L \) increases by one unit, output increases by \( K \) units. A similar argument can be used to show that \( MP_K = L \).

The MRTS is \(-K/L\).
3. The marginal product of the 5th unit of labor is 25.

Diminishing marginal returns set in with the 5th unit of labor. That is where the MP\(_L\) begins to fall.

The AP\(_L\) when the firm uses L = 8 is 16 \((128/8)\).
Costs

Practice Problems

1. Suppose a firm’s production function is given by \( Q = AL^\alpha K^\beta \). Thus the marginal product of labor is given by

\[ MP_L = \alpha AL^{\alpha-1} K^\beta \]

and the marginal product of capital is given by:

\[ MP_K = \beta AL^\alpha K^{\beta-1} \]

Suppose

\[ A = 1 \]

\[ \alpha = \frac{1}{2} \]

\[ \beta = \frac{1}{2} \]

Also, the price of labor, \( w = 16 \) and the price of capital, \( r = 9 \).

How much labor and capital should the firm hire if it wants to produce 48 units of output while minimizing its cost of production?

2. Suppose a firm’s production function is given by \( Q = L + K \). If the price of labor, \( w = 5 \) and the price of capital, \( k = 10 \), how much labor and capital should the firm hire if it wants to produce 20 units of output while minimizing its cost of production? (Hint: draw the firm’s isoquant for 20 units of production.)

3. Suppose in the short run a firm’s production function is given by \( Q = AL^\alpha K^\beta \). Since it is in the short run, capital is constant at the value \( K = 25 \). Also, suppose

\[ A = 1 \]

\[ \alpha = \frac{1}{2} \]

\[ \beta = \frac{1}{2} \]
Suppose the price of labor, \( w = 5 \). What is the marginal cost of production when the firm is producing 15 units of output? (Hint: Remember the formula we derived in class for the MC in terms of \( w \) and \( MP_L \).)

4. Complete the following table:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>TFC</th>
<th>TVC</th>
<th>TC</th>
<th>AFC</th>
<th>AVC</th>
<th>ATC</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
<td>0</td>
<td>300</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>30</td>
<td></td>
<td>300</td>
<td>30</td>
<td>330</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>50</td>
<td>350</td>
<td>150</td>
<td>25</td>
<td>175</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>60</td>
<td>360</td>
<td>100</td>
<td>20</td>
<td>120</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>100</td>
<td>400</td>
<td>75</td>
<td>25</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>220</td>
<td>520</td>
<td>60</td>
<td>44</td>
<td>104</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>450</td>
<td>750</td>
<td>50</td>
<td>75</td>
<td>125</td>
<td>230</td>
</tr>
</tbody>
</table>

Answers

1. To answer this question, set the MRTS equal to the slope of the isocost line. This will allow you to find one input in terms of the other. So for example, suppose you solve for \( K \) in terms of \( L \). Substitute this expression for \( K \) back into the production function to solve for the value of \( L \). Once you know \( L \), it is easy to find \( K \).

\[ L^* = 36, \ K^* = 64 \]

2. This is an example of perfect substitutes in production. Therefore, the firm will use whichever input is cheaper. In this case, the firm will use 20 units of labor and 0 units of capital.

3. The formula we derived in class is \( MC = \frac{w}{MP_L} \). We know that \( MP_L = a L^\alpha K^\beta \). In this case this equals \( \frac{K^{\frac{3}{2}}}{2L^2} \). We are given \( w \) and \( K \), so we need to find \( L \). We can do this from the production function. We are given \( Q = 15 \), and we know \( K = 25 \), so we can solve for \( L = 9 \). Substituting into \( MP_L \) we find \( MP_L = \frac{5}{6} \). If \( w = 5 \), then \( MC = 6 \).

4.
Practice Problems

1. Suppose a firm has fixed costs in the short run of 100. Also, its variable costs are given by VC = 4q^2. The firm’s marginal cost is MC = 8q. What is the break-even price for the firm?

2. Suppose there are n identical firms in a market. Each firm’s cost function is given by C = 25 + q^2, where q is the amount that an individual firm produces. This means that an individual firm’s marginal cost is given by MC = 2q. Also, the market demand is given by P = 100 – 2Q, where Q is the total amount of the good produced by all of the firms combined. Therefore, Q = n*q.

   a) How many firms will there be in long run equilibrium?
   
   b) How much output will each of them produce?
   
   c) What will be the market price?

3. Repeat the problem above when

   C = 36 + 4q^2
   
   MC = 8q
   
   Demand: P = 900 – 4Q.

Answers

1. The break-even price for the firm is the minimum of Average cost. This occurs where marginal cost equals average cost. Average cost is TC/q where q is the quantity of output. Total cost is the sum of fixed cost and variable cost, so average cost equals

   \[
   \frac{100 + 4q^2}{q}
   \]

   Setting this equal to 8q and solving for q, q = 5. Therefore, marginal cost equals average cost = 40. So 40 is the break-even price. So if the price is above 40, the firm will make a profit. If it is below 40, the firm will make a loss.

2. a) 9; b) 5; c) 10

3. a) 73; b) 3; c) 24
Monopoly

Practice Problems

1. Suppose a monopolist faces the following demand curve:
   
   \[ P = 4000 - 20Q \]. If the long run marginal cost of production is constant and equal to $40,

   A) What is the monopolist’s profit maximizing level of output?

   B) What price will the profit maximizing monopolist charge?

   C) How much profit will the monopolist make if she maximizes her profit?

   D) What would be the value of consumer surplus if the market were perfectly competitive?

   E) What is the value of the deadweight loss when the market is a monopoly?

   F) What is the value of the Lerner Index?

   G) What is the price elasticity of demand at the profit maximizing level of output? (Hint: you can get this from the Lerner Index or you can calculate it the way we did earlier in the course.)

2. Repeat the problem above when the demand curve is \( P = 600 - 15Q \), and the long run marginal cost of production is constant and equal to $30.

3. (Increasing Marginal Cost) Suppose a monopolist faces the following demand curve:
   
   \[ P = 200 - 4Q \]. Also, the long run total cost of the monopolist is given by \( 20Q + .5Q^2 \). Therefore, long run marginal cost is \( 20 + Q \).

   A) What is the monopolist’s profit maximizing level of output?

   B) What price will the profit maximizing monopolist charge?

   C) What would be the value of consumer surplus if the market were perfectly competitive?

   D) What is the value of producer surplus under perfect competition?

   E) What is the value of consumer surplus under monopoly?

   F) What is the value of producer surplus under monopoly?

   G) What is the value of the deadweight loss when the market is a monopoly?

   H) What is the value of the Lerner Index?

   I) What is the price elasticity of demand at the profit maximizing level of output? (Hint: you can get this from the Lerner Index or you can calculate it the way we did earlier in the course.)
Answers

1. A) \( Q^M = 99 \)
   B) \( P^M = $2020 \)
   C) Profit = $196,020
   D) CS = $392,040
   E) DWL = $98,010
   F) .98019801……..
   G) -1.0202……..

2. A) \( Q^M = 19 \)
   B) \( P^M = $315 \)
   C) Profit = $5415
   D) CS = $10,830
   E) DWL = $2707.5
   F) .904761904761……..
   G) -1.105263157894736842……..

3. A) \( Q^M = 20 \)
   B) \( P^M = $120 \)
   C) \( CS^C = $2592 \)
   D) \( PS^C = $648 \)
   E) \( CS^M = $800 \)
   F) \( PS^M = $1800 \)
   G) DWL = $640
   H) \( \frac{2}{3} \)
   I) \( -\frac{3}{2} \)
Cournot Duopoly

Practice problems

For each of the following situations, assume there are two identical firms in the market. Each firm simultaneously chooses an output level: firm 1 chooses $q_1$, and firm 2 chooses $q_2$. Thus, the market quantity is $Q = q_1 + q_2$. The market price is then determined by the demand curve. Also, if $P = a - bQ$, then firm 1’s marginal revenue is $a - 2bq_1 - bq_2$, and firm 2’s marginal revenue is $a - bq_1 - 2bq_2$. Calculate:

a) The quantity that each firm produces.

b) The market price.

c) The deadweight loss (hint: you will need to calculate total surplus if the market were competitive and total surplus when the market is a duopoly. The DWL is the difference between these two).

d) The price and quantity if the two firms merged into a monopoly.

1) Demand: $P = 500 - 3Q$; $MC = 50$

2) Demand: $P = 200 - 5Q$; $MC = 50$

Answers

1.a) $q_1 = q_2 = 50$

b) $P = 200$.

c) Total surplus under a competitive market structure is equal to $33,750$. To get this, remember that the entire surplus in a competitive market is consumer surplus. So it is just the area under the demand curve and above the price. Under the Cournot duopoly, consumer surplus is equal to $15,000$. Also, each firm gains producer surplus (profit) of $7,500. Therefore the total surplus is $30,000$. Therefore, the deadweight loss is equal to $3,750$.

d) Monopoly outcome: $Q = 75$ and $P = 275$.

2.a) $q_1 = q_2 = 10$.

b) $P = 100$.

c) Total surplus under a competitive market structure is equal to $2,250$. To get this, remember that the entire surplus in a competitive market is consumer surplus. So it is just the area under the demand curve and above the price. Under the Cournot duopoly, consumer surplus is equal to $1,000$. Also, each firm gains producer surplus (profit) of $500. Therefore the total surplus is $2,000$. Therefore, the deadweight loss is equal to $250$.

Game Theory

Practice Problems

1. Suppose that two players are playing the following game. Player A can either choose Top or Bottom, and Player B can either choose Left or Right. The payoffs are given in the following table:

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>Left</td>
</tr>
<tr>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Bottom</td>
<td>10</td>
</tr>
</tbody>
</table>

where the number on the left is the payoff to Player A, and the number on the right is the payoff to Player B.

a) Does either player have a dominant strategy, and if so what is it?

b) What is the Nash Equilibrium in pure strategies?

c) Suppose the game is played so that player A moves first and player B moves second? Using the backward induction method shown in class, what will be the outcome of the game? (Hint: it will be helpful to draw the game tree).

2. Suppose that two players are playing the following game. Player A can either choose Top or Bottom, and Player B can either choose Left or Right. The payoffs are given in the following table:

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>Left</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Bottom</td>
<td>20</td>
</tr>
</tbody>
</table>

where the number on the left is the payoff to Player A, and the number on the right is the payoff to Player B.

a) Does either player have a dominant strategy, and if so what is it?

b) What is the Nash Equilibrium in pure strategies?
c) Suppose the game is played so that player A moves first and player B moves second? Using the backward induction method shown in class, what will be the outcome of the game? (Hint: it will be helpful to draw the game tree).

Answers

1. a) Player B has a dominant strategy which is to play Right.

b) The Nash Equilibrium in pure strategies is Bottom/Right.

c) Bottom/Right will be the outcome.

2. a) Neither player has a dominant strategy.

b) There is no Nash Equilibrium in pure strategies.

c) Bottom/Left will be the outcome.